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Eternal black holes in anti-de Sitter

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Abstract: We propose a dual non-perturbative description for maximally extended Schwarzschild Anti-de-Sitter spacetimes. The description involves two copies of the conformal field theory associated to the AdS spacetime and an initial entangled state. In this context we also discuss a version of the information loss paradox and its resolution.

Keywords: AdS-CFT and dS-CFT Correspondence, Black Holes, String Duality.

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Contents

1. Introduction

An eternal black hole has an extended Penrose diagram which is depicted in figure 1. This Penrose diagram has two asymptotically AdS regions. From the point of view of each of these regions the other region is behind the horizon. It is a time dependent spacetime since there is no global timelike isometry. The regions close to the spacelike singularities can be viewed as big-bang or big-crunch cosmologies (which are homogeneous but not isotropic). We will propose that this spacetime can be holographically described by considering two identical, non-interacting copies of the conformal field theory and picking a particular entangled state. This point of view is based on Israel's description of eternal black holes [1]. A similar observation in the context of AdS/CFT was made in [2, 3, 4].[1] Here we will emphasize that by including both copies we naturally get a description of the interior region of black holes, including the region near the singularities. This holographic description can be viewed as a resolution of the initial and final singularities.

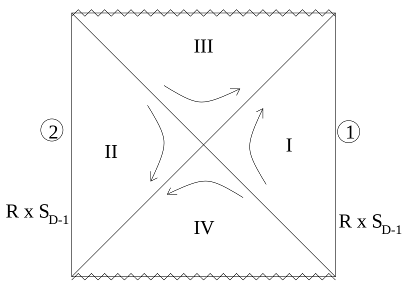

Figure 1: Penrose diagram of the extended AdS Schwarzschild geometry. Region I covers the region that is outside the horizon from the point of view of an observer on the right boundary. Region II is an identical copy and includes a second boundary. Regions III and IV contain spacelike singularities. The diagram shows the time and radial directions, over each point there is a sphere S^{d-1}. This sphere shrinks as we approach the singularities.

Using this correspondence we can study some aspects of the information loss paradox. We will formulate a precise calculation on the eternal black hole spacetime of figure 1. The result of this calculation shows information loss. We will show that information can be preserved after summing over geometries.

[1]In [3] the formula for the entangled state as a function of the temperature is off by a factor of 2. It seems to be the related to the factor of 2 that led to the claim [5] that the black hole temperature is twice what Hawking originally computed.

2. The correspondence

We start with an AdS_{d+1} spacetime and its holographic dual conformal field theory CFT$_d$, as in [6, 7, 8] (for a review see [9]). The conformal field theory is defined on a cylinder $R \times S^{d-1}$. This cylinder is also the boundary of AdS. A general conclusion of the studies of AdS/CFT is that the boundary conditions in AdS specify the theory and the normalizable modes in the interior correspond to states [10, 9]. When we give a particular spacetime which is asymptotically AdS we are giving a CFT and a particular state in the CFT.

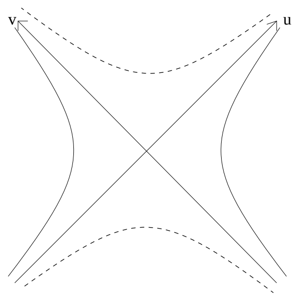

Figure 2: The lorentzian black hole in Kruskal coordinates. The singularity is at $uv = 1$ and the boundary of AdS is at $uv = -1$.

We consider the so called "big" black holes in AdS. These are black holes which have positive specific heat and are the dominant contribution to the canonical thermodynamic ensemble. In our analysis below we will write explicit formulas for black holes in AdS_3 since calculations are easiest in that case, but everything we say also holds for big black holes in AdS_{d+1}, $d \geq 2$. The euclidean metric of three dimensional black hole can be written in the following equivalent forms [11]

$$\begin{aligned}
ds^2 &= \left(r^2-1\right) d\tilde{\tau}^2 + \frac{dr^2}{r^2-1} + r^2 d\tilde{\phi}^2 \\
ds^2 &= 4\frac{dzd\bar{z}}{(1-|z|^2)^2} + \frac{(1+|z|^2)^2}{(1-|z|^2)^2} d\tilde{\phi}^2 \\
\tilde{\phi} &= \frac{2\pi\phi}{\beta}, \qquad \tilde{\tau} = \frac{2\pi\tau}{\beta}, \qquad z = |z|e^{i\tilde{\tau}} \\
\phi &= \phi + 2\pi\,, \qquad \tau = \tau + \beta
\end{aligned} \tag{2.1}$$

where we have set $R_{AdS} = 1$. τ, ϕ are to be thought of as the coordinates of the space on which the CFT is defined so that β^{-1} is the temperature. Notice that the boundary of AdS is at $|z| = 1$. By analytically continuing in the imaginary part of z in (2.1), and setting $z = -v, \bar{z} = u$, we obtain the eternal black hole in Kruskal coordinates

$$ds^2 = \frac{-4dudv}{(1+uv)^2} + \frac{(1-uv)^2}{(1+uv)^2} d\tilde{\phi}^2 \tag{2.2}$$

where $u = t + x, v = t - x$. In these coordinates the spacetime looks as in figure 2. The event horizons are at $u = 0$ and $v = 0$. The boundary of AdS is at $uv = -1$ and the past and future singularities are at $uv = 1$. In these coordinates it is clear that nothing special happens at the horizon. Notice that the metric (2.2) is time dependent. It has the boost-like isometry that acts by $u \to e^{\lambda}u$, $v \to e^{-\lambda}v$. This is the usual "time" translation invariance in Schwarzschild coordinates. The orbits of these isometries are time-like going forward in time in region I, timelike going backwards in time in region II and spacelike in regions III and IV, see figure 1. The metric (2.2) has a reflection symmetry under $t \to -t$. In fact, we can glue the $Im(z) = 0$ cross section of the euclidean metric (2.1), to the

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$t = 0$ spatial cross section of (2.2). We can view the euclidean part of the geometry as giving the initial wavefunction which we then evolve in lorentzian signature. This is the Hartle-Hawking construction of the wavefunction [12].

Let us understand what this wavefunction is from the point of view of the boundary CFT. The boundary of the euclidean black hole is $S^1_\beta \times S^{d-1}$. The section of the euclidean metric (2.1) at $Im(z) = 0$ intersects the boundary on two disconnected spheres $S^{d-1} + S^{d-1}$. (In the three dimensional case (2.1) we get two circles $S^1_\phi + S^1_\phi$). The euclidean time direction connects these two spheres. The path integral of the boundary CFT is then over $I_{\beta/2} \times S^{d-1}$, where $I_{\beta/2}$ is an interval of length $\beta/2$. This path integral then gives a wavefunction on the product of two copies of the CFT. Let $\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2$ denote the full Hilbert space consisting of two copies of the Hilbert space of the CFT. The wavefunction $|\Psi\rangle \in \mathcal{H}$ is

$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2 \,. \qquad (2.3)$$

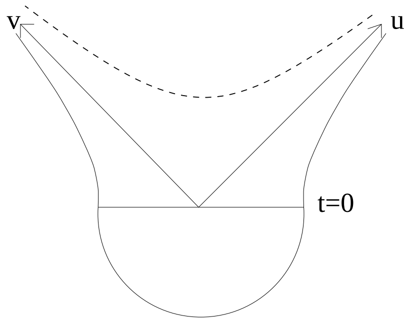

Figure 3: The Hartle-Hawking-Israel wavefunction can be thought of as arising from gluing half the euclidean geometry at $t = 0$ to half the lorentzian geometry. Over each point on this diagram there is a sphere S^{d-1}.

The sum runs over all energy eigenstates the system and the subindex 1,2 indicates the Hilbert space where the state is defined. $Z(\beta)$ is the partition function of one copy of the CFT at temperature β^{-1} and it is necessary in (2.3) for normalization purposes. We view this wavefunction as an initial condition for lorentzian evolution. More precisely, this is the wavefunction at lorentzian time $t_1 = t_2 = 0$. We have two times since we have two independent copies of the field theory and we can evolve as much as we want in each time. Note, however, that the state $|\Psi\rangle$ is invariant under $\tilde{H} = H_1 - H_2$. This construction with two copies of a field theory in a pure state given by (2.3) is very familiar in the description of real time thermal field theories and goes under the name of "thermofield dynamics" [13]. Thermal expectation values in any field theory can be calculated as conventional quantum mechanical expectation values in two copies of the field theory in the inital pure state (2.3). In other words

$$\langle\Psi|\mathcal{O}_1|\Psi\rangle = Tr[\rho_\beta \mathcal{O}_1] \qquad (2.4)$$

where $\mathcal{O}_1$ is any operator defined on the first copy of the field theory. Since the left hand side of (2.4) contains no operators acting on the second copy of the field theory we can sum over all states of the second copy of the field theory and obtain the result on the right hand side of (2.4). After doing this sum we get the thermal density matrix in the first copy of the field theory. One views the thermal density matrix as arising from entanglement. The entropy is the entanglement entropy [13].

The proposal is that *two copies of the CFT in the particular pure (entangled) state (2.3) is approximately described by gravity on the extended AdS Schwarzschild spacetime.* The meaning of the word "approximately" will become clear later. The "boost" symmetry of

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the AdS-Schwarzschild spacetime is the same as the symmetry under $\tilde{H}$ of the two copies of the CFT and the state (2.3). If we do not do any observations in the second copy of the conformal field theory, i.e. we do not insert any operators on the second boundary of the AdS Schwarzschild spacetime, then expectation values on the first copy become thermal expectation values. The connection between the extended Schwarzschild geometry and "thermofield dynamics" was first noticed by Israel [1], see also [14, 15]. Here we are just pointing out that that in the context of AdS-Schwarzschild geometries this connection becomes precise and that it gives, in principle, a way to describe the interior. The fact that the eternal black hole in AdS is related to an entangled state in the CFT was observed in [2, 3]. The fact that black hole entropy and entanglement entropy are related was observed in [16, 17].

In the semiclassical approximation, besides giving a geometry, we need to specify the state of all fields living on this geometry. In general time dependent geometries there is no obvious way to construct the state. In our case the state of the field is fixed by patching the euclidean solution as described above which gives the standard Hartle-Hawking state. One can also specify the state of a quantum field on a general background by specifying the set of positive energy wavefunctions, see [18]. The Hartle-Hawking state is obtained if one defines this set to be the set of wavefunctions which restricted to $u = 0$ ($v = 0$) have an expansion in terms of $e^{-i\omega v}$ ($e^{-i\omega u}$) with $\omega > 0$. This Hartle-Hawking-Israel state is such that the expectation value of the stress tensor is non singular (except at the past and future singularities).

In the AdS_3 case it is very easy to construct this state since the BTZ black hole is a quotient of AdS_3. This quotient is the one that makes ϕ periodic [11]. If ϕ were non-compact we would have an infinite non extremal black string, whose extended Penrose diagram is global AdS. It can be seen that the H-H prescription of gluing half the euclidean solution coincides with the prescription that gives the global AdS vacuum, which is to glue in half of an infinite euclidean cylinder. In both cases we glue in half a three ball. This implies that the notion of positive frequency as defined in global AdS coincides with the notion of positive frequency defined in the Hartle-Hawking vacuum of the infinite black string.[2] By doing the quotient we restrict the set of possible wavefunctions but we do not change the notion of positive frequencies. This implies that we can obtain Green's functions on the Hartle-Hawking state by taking the usual Green's functions on global AdS and adding over all the images under the group that generates the quotient. For Schwarzschild-AdS black holes in other dimensions one would have to work harder but the procedure is a straight forward analytic continuation from the euclidean solution.[3]

Now that we have specified the state we can compute, in the semiclassical approximation, the correlation functions for insertions of operators on various boundaries. Let us consider a scalar field in AdS_3 which corresponds to a CFT operator of dimension $(L_0, \bar{L}_0) = (\Delta, \Delta)$. By summing over all images we obtain the time ordered correlator for

[2]This is related to the observation in [19] this non-extremal black string can be understood as the usual CFT on R^2 in the usual Minkowski vacuum but viewed in Rindler space.

[3]Vacua for black holes in AdS_2 were discussed in [20].

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two operators inserted on the same boundary [21]

$$\langle \Psi | T(\mathcal{O}_1(t,\phi)\mathcal{O}_1(0,0)) | \Psi \rangle \sim \sum_{n=-\infty}^{\infty} \frac{1}{\left[\cosh(\frac{2\pi t}{\beta}) - \cosh(\frac{2\pi(\phi+2\pi n)}{\beta}) - i\epsilon\right]^{2\Delta}} . \tag{2.5}$$

For operators inserted on opposite boundaries we obtain

$$\langle \Psi | \mathcal{O}_1(t_1,\phi_1)\mathcal{O}_2(t_2,\phi_2) | \Psi \rangle \sim \sum_{n=-\infty}^{\infty} \frac{1}{\left(\cosh(\frac{2\pi(t_1+t_2)}{\beta}) + \cosh(\frac{2\pi(\phi_1-\phi_2+2\pi n)}{\beta})\right)^{2\Delta}} \tag{2.6}$$

where we have defined time on the second copy of the CFT so that it increases towards the future. The reason that we get a non-vanishing correlator despite the fact that the operators live in two decoupled field theories is due to the fact that we have an entangled state, as in the EPR experiment. Operators on different boundaries commute.

It is easy to see how to insert particles in the interior of the extended AdS-Schwarzschild spacetime. A particle is a small fluctuation around some state, it is a small deformation of the wavefunction. The H-H wavefunction is defined by doing the path integral over half the euclidean geometry. We can generate the H-H state plus some particles by gluing the same euclidean geometry but now with some operators inserted at the boundary, see figure 4. This deforms slightly the wavefunction and we will not have the state (2.3) but a slightly different one. In the case of AdS_3 we can do this explicitly and say that an operator inserted at a point $(z_0 = e^{i\theta_0}, \phi_0)$ along the euclidean boundary (with $-\pi \leq \theta_0 \leq 0$) creates a particle with the lorentzian wavefunction

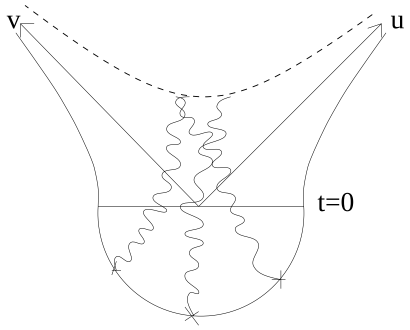

Figure 4: We can add particles to the Hartle-Hawking-Israel state by acting with operators on the euclidean field theory.

$$\phi(t,x,\phi) \sim \frac{(1+uv)^{2\Delta}}{\left\{(1-uv)[\cosh(\frac{2\pi(\phi-\phi_0+2\pi n)}{\beta}) - 1] + (u - e^{-i\theta_0})(v + e^{i\theta_0})\right\}^{2\Delta}} . \tag{2.7}$$

It is easy to see that this wavefunction has positive frequency, in the Hartle-Hawking sense. We could form other wavefunctions by convoluting the operator with appropriate functions of the boundary point. In principle we can also insert particles by acting with operators in the far past in the lorentzian description. From the CFT point of view this is clear. The fact that we can create particles in region IV (see figure 1) by insertion of boundary operators is essentially the idea of complementarity [22].

Note that these particles are created by operators that are acting on both copies of the field theory. As a simple example, let us replace the CFT by a single harmonic oscillator. The state (2.3) becomes

$$|\psi\rangle = \frac{1}{\sqrt{Z}} e^{e^{-\beta w/2} a_1^\dagger a_2^\dagger} |0\rangle . \tag{2.8}$$

This state is related by a Bogoliubov transformation to the vacuum so that it looks like the vacuum for the oscillators

$$\begin{aligned} \tilde{a}_1^\dagger &= \cosh\theta a_1^\dagger - \sinh\theta a_2 \,, \qquad \tilde{a}_2^\dagger = \cosh\theta a_2^\dagger - \sinh\theta a_1 \\ \tilde{a}_i|\psi\rangle &= 0 \,, \qquad \tanh\theta = e^{-\beta w/2} \,. \end{aligned} \tag{2.9}$$

We conclude that particles are created by the oscillators $\tilde{a}_i^\dagger$ and these involve operators on both decoupled theories.

It is also easy to see that the expectation value of the stress tensor for quantum fields in the BTZ geometry diverges as we approach the singularities [23, 24]. If we define the stress tensor via a point splitting procedure, as explained in [18], then we will need to compute the bulk-bulk two point function for fields in the interior. The two point function can be obtained by summing over all the images. Away from the singularity the images are spacelike separated. As we approach the singularity the images become light-like separated and on the other side they would be timelike separated. The divergence of the stress tensor is just due to the standard divergence of the two point function at lightlike separated points.

A similar argument shows that for the rotating black hole in AdS_3, i.e. the black hole with angular momentum along the ϕ circle, the quantum stress tensor is singular at the inner horizon [25]. The killing vector associated to the identification becomes lightlike at the singularity. The generator of the identification is obtained by exponentiating the action of this Killing vector. Once we exponentiate it is possible to get images that are lightlike separated before we get to the singularity, in fact we get them as soon as we cross the inner horizon. This does not contradict the statement in [11] that there are no closed timelike curves within the fully extended Penrose diagram. What happens is that a locally timelike curve joining two timelike separated images goes through the so called "singularity". Due to this divergence of the stress tensor we can only trust the geometric description up to the inner horizon.[4]

These eternal black holes with angular momentum along the ϕ circle are dual to the same two copies of the CFT but now instead of the state (2.3) we have

$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta,\mu)}} \sum_n e^{-\frac{\beta E_n}{2} - \frac{\beta\mu\ell_n}{2}} |E_n, \ell_n\rangle_1 \times |E_n, \ell_n\rangle_2 \tag{2.10}$$

where μ is the chemical potential for momentum along the circle and ℓ_n denotes the angular momentum of the state. We can do a similar construction for any other conserved charges of the CFT to describe charged black holes in AdS, see [1].

Given that the boundary description, in principle, describes the interior one would like to give a precise prescription for recovering the approximately local physics in the interior, i.e. we should be able to describe approximately scattering amplitudes measured by an observer who is behind the horizon and falling into the singularity. If interactions between bulk particles are weak it is easy to give a prescription. We can map initial and final states to states in the CFT as we described above and then we can compute the overlap between initial and final states in the CFT. This can be viewed as mapping all states to the

[4] A related remark was made in [26].

$t = 0$ slice and computing the inner product there. This is also equivalent to computing amplitudes by analytically continuing to euclidean space in the way we explained above. If interactions are strong the particles will scatter many times before we evolve back to $t = 0$. Though in principle we can do the computation it would be very hard in practice to extract the desired amplitude. This problem is present not only for black hole spacetimes but also in usual AdS/CFT. It is the problem of extracting local bulk physics from the CFT. The mapping between the state in the CFT and multiparticle states in the bulk can be very complicated, see for example [27]. Certain situations, where particles get well separated from each other after a time of order the AdS radius can be described as explained in [28]. On the other hand if we have a large number of particles with multiple interactions within an AdS radius then it becomes complicated to state how to recover local computations in the interior.

As an aside, let us note that another situation with two boundaries is the case of AdS_{d+1} space written with AdS_d slices. The system in this case is dual to two field theories defined on AdS_d which are coupled by their boundary conditions on the boundary of AdS_d (see [29]). Indeed, in the bulk spacetime one can send signals between the two AdS_d boundaries. This is different from the situation we considered above, where the two field theories where decoupled. In our case we do not expect to be able to send signals between the two boundaries and indeed in the geometry we find that they separated by a horizon.

If we start with a CFT with only one connected boundary we cannot get geometries with two disconnected boundaries because they would have infinite action. When we specify the CFT and say on which space it lives we are implicitly giving a set of counterterms for the gravity solution. If we start with only one boundary then there could exist geometries which have additional boundaries but they will have infinite action and will not contribute to the computation if we do not include the counterterms for the extra components of the boundary. These counterterms are local and depend only on the asymptotic structure of the solution [8, 30], but we need to say over which surfaces they are integrated. This choice of surface is the choice of space over which the field theory is defined. Notice in particular that we are *not* interpreting figure 3 as a gravitational instanton giving the amplitude to create two boundaries. The action of the euclidean instanton is infinite, if we do not include the regularizing counterterms on the euclidean boundary. Including the boundary conterterms is specifying precisely what theory we have, it is saying that we have precisely the euclidean field theory on a very specific geometry. The process depicted in figure 3 should be thought of as the process that prepares the entangled state, both in field theory and in gravity.

If the curvature of the boundary is positive and we are in euclidean space it was shown in [31] that the boundary cannot have disconnected pieces. If the boundary has negative curvature one can have several disconnected pieces. In the case of AdS_3 we expect to be able to consider the field theory on negatively curved Riemman surfaces as long as we are not at a singular point of the CFT [32]. In this situation it would be interesting to understand the meaning of euclidean geometries with disconnected boundaries.

Finally let us notice that in the case of AdS_3 there are many interesting lorentzian spacetimes that have multiple boundaries [33]. These can be obtained by gluing suitable euclidean geometries [34]. These geometries also have the interpretation of being several copies of the CFT in a entangled state that can be obtained by doing the path integral of the euclidean theory on the boundary of the euclidean geometry. In lorentzian signature some of these spacetimes have one boundary and some have multiple boundaries.

It seems that all lorentzian spacetimes with multiple boundaries can be thought of as entangled states arising in the product Hilbert space of many decoupled conformal field theories [35]. In fact, it was shown in [35, 36] that in lorentzian spacetimes obeying the null positive energy condition all boundaries are screened from other boundaries by horizons. This is enough to ensure the vanishing of commutators for operator insertions on different boundaries, at least to leading order in the gravity approximation.

2.1 Black holes with only one boundary[5]

One can take quotients of the eternal black hole spacetime in figure 1, and obtain black hole spacetimes with only one boundary. The main idea is to quotient by a map that acts as a reflection $x \to -x$ on the Kruskal coordinates (2.2) (in other words $u \leftrightarrow v$). This action can be accompanied by many other Z_2 actions on the full theory. One possibility, which was discussed in detail in [37], is to also map a point on S^{d-1} to its antipodal point, this has the advantage of being a non-singular quotient. In the full string theory one might need to accompany this action with an orientifold action, etc (see [38] for an example). These black holes are particular pure states in one copy of the conformal field theory that lives on the boundary of AdS_{d+1}. By thinking about patching a euclidean solution at a moment of time reversal symmetry it is easy to construct these states, both the state in the boundary CFT and the state for quantum fields in the bulk.

Let us start with the CFT_d on a space which is $(S^1_\beta \times S^{d-1})/Z_2$ where Z_2 acts by mapping a point on S^{d-1} to its antipodal point and by a reflection $(\tau \to -\tau)$ on the circle S^1_β. The length of S^1_β before we do the quotient is β. The geometry thus obtained is non-singular, it is a "Klein bottle" (it is the Klein bottle for $d = 2$). If we cut this euclidean geometry at $\tau = \beta/4$, were τ is a point on S^1_β, we get two copies of a "Moebius strip" (it is the usual Moebius strip for $d = 2$). The boundary of this "strip" is S^{d-1}. So the path integral of the euclidean theory over this "strip" produces a state of the CFT on S^{d-1}. The $d = 2$ version of this construction is very familiar, we produced the crosscap boundary state evolved by $\beta/4$ with the closed string hamiltonian, which is a state in the closed string Hilbert space. In summary, we have produced the state of the lorentzian CFT at $t = 0$ by a euclidean path integral over a "Moebius strip".

Now let us discuss the dual gravity description. Let us start with the euclidean theory. There are (at least) two ways of filling in the Klein bottle which are related to the two ways of filling in $S^1_\beta \times S^{d-1}$ [39]. One way of filling it in is with a space of topology $D^2 \times S^{d-1}$, this is the euclidean Schwarzschild AdS black hole. The Z_2 quotient gives a non-singular space. This geometry has a time reflection symmetry at $\tau = \beta/4$. The spatial geometry of this slice is that of the $t = 0$ section of the quotiented Kruskal diagram of a black hole in AdS_{d+1}, so we can patch the lorentzian solution to the euclidean solution.

[5] This subsection originated in conversations with G. Horowitz.

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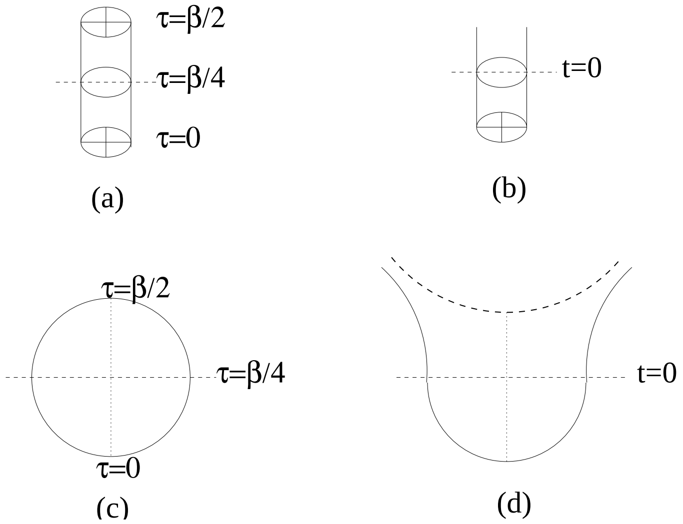

Figure 5: Z_2 quotients of the eternal black hole. In (a) we see the Z_2 quotient of the CFT. It is a euclidean cylinder going between two boundary states. These could be cross caps, so that we get a "Klein bottle". In (b) we cut (a) along the doted lines and view the resulting state as the $t = 0$ quantum state which is then evolved using lorentzian evolution. In (c, d) we see the bulk gravitational description of (a, b). The vertical doted line indicates that the left and right sides are identified. In (c) the boundary cylinder of (a) is represented as the arc going from $\tau = 0$ to $\tau = \beta/2$.

This is represented pictorially in figure 5(d). As we explained above for the eternal black hole, this construction determines, in a precise way, the quantum state for the fields in the lorentzian solution. Particles in the interior are obtained by inserting operators in the euclidean geometry as in figure 4. Some aspects of this quantum state were discussed in [37]. In the AdS_3 case the supergravity correlators were computed in [37] and are given, again, by the method of images, except that now we have to include images under the Z_2 action also. This implies that we essentially have to add (2.5) and (2.6) together.

We can consider other Z_2 quotients. For example, we can choose a Z_2 which purely reflects the euclidean time direction and does not act on the sphere. This Z_2 action has fixed point on the boundary. In fact, this can be interpreted as a conformal field theory with a boundary. All we have to do is to substitute the crosscap in the above discussion, and in figure 5, by a usual Cardy boundary state.

In all of our discussion in this section we have assumed that we can actually do this Z_2 orbifold. Both in the field theory and in string theory we have to be careful about the presence of spinors, so that the Z_2 will also have to act on them appropriately. In order to make the discussion more concrete let us mention two examples of Z_2 actions we could consider. As an example where the Z_2 action does not have fixed points on the boundary consider $AdS_3 \times S^3 \times T^4$. The Z_2 action is an orientifold together with the following geometrical action the euclidean time direction, it shifts the circle S^1_ϕ by π (this is the antipodal map for the circle) and it sends $g \to g^\dagger$, where $g \in \mathrm{SU}(2)$ describes the S^3.

As an example of a Z_2 action with fixed points on the boundary consider the following. Start from the field theory that results from taking the low energy limit of coincident M2 branes which is dual to $AdS_4 \times S^7$. Take the Z_2 to be a reflection on S^1_β as in [40]. This introduces a boundary in the euclidean field theory, the M2 branes can end on this boundary. The state is prepared from the boundary state via euclidean evolution as above. In this case the vertical doted line in figure 5 is an end of the world ninebrane in 11 dimensions. In fact the M2 brane is stretched between an end of the world ninebrane (at $\tau = 0$) and an end of the world anti-ninebrane (at $\tau = \beta/2$), as in [41]. Its worldvolume is $I_{\beta/2} \times S^2$, where $I_{\beta/2}$ is an interval of length $\beta/2$.

Even though the black hole is given by a pure state we expect to find approximate thermal answers if we do measurements that involve a very small subset of degrees of freedom. When we only probe part of the system the rest of the system is acting as a thermal reservoir at temperature β.

3. Remarks about information loss

The information loss paradox [42] becomes particularly sharp in AdS because we can form black holes that exist for ever. The information loss argument in [42] says that after matter collapses into a black hole all correlators with the infalling matter decay exponentially. In [12] it was argued that computations done at late times will be the same as computations done in the full extended Schwarzschild geometry. These computations show that only thermal radiation comes out and therefore information gets lost. In the case of black holes that evaporate in finite time there are some residual correlations with the initial state. These correlations are of the order of $e^{-ct_{evap}/\beta} \sim e^{-c'S}$ where t_{evap} is the evaporation time for Schwarzschild black holes in flat space and c, c' are some numerical constants. In the case of black holes in AdS the black hole lives for ever so one can wait an arbitrary long time for correlations with the initial state to decay.[6]

Here we will consider the simplest possible deformation of the perfectly thermal Schwarzschild AdS state and we will show that, indeed, correlations die off exponentially fast. The simplest possible deformation of the thermal state is to add an operator in the second boundary. From the point of view of the first boundary this is a small change in the thermal ensemble. This change is detectable. Indeed we can compute the one point function of the same operator in theory one. This correlator is zero in the perfectly thermal ensemble (when there is no operator on boundary two), but it is non-zero in the deformed ensemble. This non-zero value is given by (2.6). In agreement with the arguments in [42, 12], this correlator decays exponentially fast as $e^{\frac{-ct}{\beta}}$ where c is a numerical constant. If we wait a sufficiently long time this correlator goes to zero. This is *not* what we expect if we make a small change of the density matrix in a unitary theory, such as the boundary CFT. In some sense, we can say that the extended AdS Schwarzschild spacetime is "more thermal" than a thermal state in a unitary theory. This is a version of the information loss paradox. It is a particularly sharp version of the paradox since the calculation is very well defined.

It has been suggested that string theory, being a theory of extended objects, invalidates arguments based on local field theory, such as the arguments that lead to information loss.

[6]For other discussions of information loss in AdS/CFT see [43, 44].

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It is therefore natural to ask if these effects could solve the paradox that we have just presented. The BTZ black hole in AdS_3 is particularly useful to test this idea. We can embed AdS_3 in string theory in such a way that strings moving in AdS_3 are described by an SL$(2,\mathbb{R})$ WZW model. Two point correlation functions in AdS_3 computed using string theory have the same functional form as in field theory [45, 46]. This is easy to understand since both are restricted by conformal invariance (i.e. global SL$(2,\mathbb{R})^2$ invariance). The BTZ black hole is a quotient of AdS_3. The standard orbifold rules imply that the correlation function is given by "summing over the images". This prescription gives the stringy version of the Hartle-Hawking state. This produces a result that has the same functional form as in field theory (2.5) (2.6) . This shows that tree level string theory does not solve the problem. One could think that higher loops in string theory would solve the problem, this might be possible, but as we will see below this expectation seems unfounded.[7]

Before we go on looking for corrections we should ask: How big are the expected correlations? We now show that a correlation that is consistent with unitarity could be as small as of order e^{-cS} where S is the entropy of the ensemble and c is a numerical constant. Instead of presenting a general argument, let us do a calculation in a free field theory and see that correlations as small as e^{-cS} are possible. The field theory we will consider is similar to the so called "long string model" of the black hole [48]. We consider a single field $X(\tau,\sigma)$ that lives on a circle of radius k where $k \sim \frac{R_{AdS}}{G_N^{(3)}}$, so that $\sigma = \sigma + 2\pi k$. The operator that we will consider is $\mathcal{O} = \sum_n \partial X \bar{\partial} X(\tau, \sigma + 2\pi n)$. We can use the standard formulas of finite temperature field theory [13] to compute the two point correlation function of an operator inserted in the second copy with an operator inserted on the first copy.

$$\langle \mathcal{O}_1(\tau,\sigma)\mathcal{O}_2(0,0)\rangle_{Free} \sim \sum_m \left[\sum_{n=-\infty}^{\infty} \frac{1}{\left(\cosh(\frac{2\pi(t+2\pi mk)}{\beta}) + \cosh(\frac{2\pi(\phi+2\pi n)}{\beta})\right)^2} \right]. \tag{3.1}$$

We see that the quantity in square brackets has the form of the gravity result (2.6) and the role of the sum over m is to make the result periodic in time, under $t \to t + 2\pi k$. This periodicity follows from the fact that all energies in this free theory are multiples of $1/k$. We notice, however, that between two maxima this function is very small, it is of order $e^{-(2\pi)^2 k/\beta} \sim e^{-cS}$ as we wanted so show. In a full interacting field theory we do not expect to find a periodic answer, but this calculation shows us that the correlations can be as small as e^{-cS}. Since the entropy is proportional to $1/G_N$ we see that these could come from non-perturbative effects, so there would be nothing wrong if we did not see any effect in string perturbation theory.

Let us return to the eternal black holes in AdS. The correlation functions in the boundary field theory clearly cannot decay to zero at large times. The problem is solved once we remember that the AdS/CFT prescription is to some over *all* geometries with prescribed boundary conditions. In particular, the euclidean thermal ensemble has other

[7]In the case of the SL$(2,\mathbb{R})$ WZW model we find that one loop corrections diverge as explained in [47]. This is related to the fact that the thermal ensemble is unstable since the black hole can evaporate by emitting long strings. In stable thermal ensembles we do not expect such a divergence.

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geometries besides the one we included so far. One of them is that of an AdS space with euclidean time periodically identified. This is a geometry with topology $S^1 \times B^d$. The fact that we should sum over geometries in the euclidean theory was emphasized in [49, 39, 19, 50]. Since we can view the euclidean path integral as defining our initial wave function we will also get other geometries that contribute to the lorentzian computation. The geometry that provides the effect that were are looking for consists of two separate global AdS spaces with a gas of particles on them. This gas of particles is in an entangled state. This piece of the wavefunction originates from a euclidean geometry which is an interval in time of length $\beta/2$ in global euclidean AdS. The euclidean evolution by $\beta/2$ is responsible for creating the entangled state for the gas of particles. Now if we compute the two point correlator in this geometry we indeed get a non-decaying answer. This geometry is contributing with a very small weight due to its small free energy (remember the factor $Z(\beta)^{-1/2}$ in (2.3)) compared to the AdS Schwarzschild geometry. In fact the size of the contribution is of order $e^{-\beta(F_{AdS}-F_{Black-hole})} \sim e^{-c'S}$ where c' is some constant. So we get the right amplitude for the non-decaying correlator.

It was argued in the past that the sum over geometries would destroy unitarity since it would involve black holes. It is amusing to note that here the sum over geometries is involved in restoring unitarity.[8] The fact that the sum over geometries can restore unitarity was observed in [51, 52].

One could ask how to restore unitarity in the case that we start with a pure state in a single copy of the field theory. In fact we can consider the Z_2 quotients we discussed above, which produce black holes with a single boundary. The two point correlation function also decays exponentially in this case. Once we remember that there are other ways of filling in the geometry we realize that we get non-decaying contributions to the correlation function.

Finally let us remark that even though we chose to compute a correlator between theory two and theory one, we could have computed a correlator between two insertions in theory one. This could be viewed as throwing in a particle in a perfectly thermal state and asking whether we can see any change in the state at late times. If we look at (2.5) carefully we can also see that these correlations decay exponentially in time (we need to convolute with a smooth function at the initial point taking into account the $i\epsilon$ prescription).

4. Conclusions

We have seen how to describe, in principle, the spacetime corresponding to extended AdS Schwarzschild geometries. More precisely, the AdS Schwarzschild geometry appears as the saddle point contribution of a more complicated sum over geometries. This gives, in principle, the resolution of the spacelike singularities in the interior of black holes. These spacetimes are very interesting since they are simple cosmological spacetimes. The resolution of the singularities in this case reduces simply to the specification of initial

[8] G. Moore reminded me of the connection between unitarity in the lorentzian theory and modular invariance in the euclidean theory. In the AdS_3 case the sum over geometries was required by modular invariance [19, 50]. In string perturbation theory this connection is well known, we see it appearing again in a (apparently) different context.

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conditions in the full system, the full system includes other classical geometries as well. There are many initial conditions corresponding to different particles coming out of the white hole singularity. The wavefunction $|\Psi\rangle$ (2.3) is a very special choice for which we have a geometric interpretation. Of course these "cosmologies" are very unrealistic since they are highly anisotropic (but notice that by considering black holes in AdS_4 they can be four dimensional). These solutions show that it is possible to have cosmological looking spacetimes in non-perturbative string theory. If we start the system in the state (2.3) and we let it evolve, after a time of order β we expect that the geometric description would break down for an important part of the state. But since the evolution is just adding phases to the state, by the quantum version of the Poincare recurrence theorem after a very long time will get arbitrarily close to the initial state and therefore we would recover the geometric interpretation, but with slightly different initial conditions, so that we will not have the H-H state but we will have the H-H state with some particles coming out of the white hole singularity. Interpreted as a cosmology, the universe gets to start over and over again and the initial conditions change slightly every time.

It would be very nice to understand more precisely how to describe local processes in the interior in terms of the boundary theory.

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